Special Practice Problems

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~[JEE (Mains & Advanced)]~

Topic: Matrices

**** Cayley–Hamilton Introduced Matrix theory (named after the mathematicians Arthur Cayley and William Rowan Hamilton)**

***If you shuffle a pack of cards properly, chances are that exact order has never been seen before in the whole history of the universe.

Objective Questions Type I [Only one correct answer]

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

1. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T(Q^{2005})P$ is equal to

(a) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{\sqrt{3}}{2} & 2005 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2005 \\ \frac{\sqrt{3}}{2} & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & \frac{\sqrt{3}}{2} \\ 0 & 2005 \end{bmatrix}$

2. What is wrong in the following computation ?

$$\begin{bmatrix} 1 & 0.01 \\ 1 & 1 \end{bmatrix}^n = \left\{\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 10^{-2}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right\}^n$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^n + n \times 10^{-2}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{n-1}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Since, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}^k = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, for $k \geq 2$.

(a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}^k = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, for $k \geq 2$ is not true

(b) Computation of second term on RHS is not valid

(c) First term should be calculated completely

(d) None of the above

3. If $A^5 = O$ such that $A^n \neq I$ for $1 \leq n \leq 4$, then $(I - A)^{-1}$ is equal to

(a) A^4

(b) A^3

(c) $I + A$

(d) none of these

4. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $\lim_{n\to\infty} \frac{A^n}{n}$ is (where $\theta \in R$)

(a) a zero matrix

(b) an identity matrix

(c) $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

5. If $A = \text{diag}(d_1, d_2, d_3, \ldots, d_n)$, then A^n is equal to

(a) $\text{diag}(d_1^{n-1}, d_2^{n-1}, d_3^{n-1}, \ldots, d_n^{n-1})$

(b) $\text{diag}(d_1^n, d_2^n, d_3^n, \ldots, d_n^n)$

(c) $\text{diag}(d_1, d_2, d_3, \ldots, d_n)$

(d) $\text{diag}(d_1, d_2^2, d_3^3, \ldots, d_n^n)$

6. The rank of the matrix $\begin{pmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{pmatrix}$ is (where $a = -6$)

(a) 1

(b) 2

(c) 3

(d) 4

7. If A is a square matrix of order 2×2 such that $A^2 = O$, then

(a) $A = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$, where α, β, γ are numbers such that $\alpha^2 + \beta\gamma = 0$

(b) $A = \begin{pmatrix} \alpha & \beta \\ \beta & -\alpha \end{pmatrix}$ with $\alpha = \pm\beta$

(c) $A = \begin{pmatrix} \alpha & -\alpha \\ -\beta & \beta \end{pmatrix}$ with $\alpha^2 + \beta^2 = 1$

(d) none of the above

8. If $2x - y = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 3 & 2 \end{bmatrix}$ and $2y + x = \begin{bmatrix} 4 & 1 & 5 \\ -1 & 4 & -4 \end{bmatrix}$, then

(a) $x + y = \begin{bmatrix} 3 & 0 & -1 \\ 0 & 3 & -2 \end{bmatrix}$

(b) $x = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

(c) $x - y = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$

(d) $y = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 1 & -2 \end{bmatrix}$

9. If $A = \begin{pmatrix} \frac{-1+i\sqrt{3}}{2i} & \frac{-1-i\sqrt{3}}{2i} \\ \frac{1+i\sqrt{3}}{2i} & \frac{1-i\sqrt{3}}{2i} \end{pmatrix}$, $i = \sqrt{-1}$ and $f(x) = x^2 + 2$, then $f(A)$ equals

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\left(\frac{3-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\left(\frac{5-i\sqrt{3}}{2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $(2+i\sqrt{3})\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

10. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $A^2 = 8A + kI_2$, then k is equal to

(a) -1 (b) 1
(c) -7 (d) 7

11. If $\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ is zero matrix, then θ and ϕ differ by

(a) even multiple of $\frac{\pi}{2}$ (b) odd multiple of $\frac{\pi}{2}$
(c) even multiple of π (d) odd multiple of π

12. Let A and B be two matrices, then

(a) $AB = BA$ (b) $AB \neq BA$
(c) $AB < BA$ (d) $AB > BA$

13. Let A and B be two matrices such that $A = O$, $AB = O$, then equation always implies that

(a) $B = O$ (b) $B \neq O$
(c) $B = -A$ (d) $B = A'$

14. In matrices

(a) $(A+B)^2 = A^2 + 2AB + B^2$
(b) $(A+B)^2 = A^2 + B^2$
(c) $(A+B)^2 \neq A^2 + 2AB + B^2$
(d) $(A+B)^2 = A^2 + 2BA + B^2$

15. The characteristic of an orthogonal matrix A is

(a) $A^{-1} \cdot A = I$ (b) $A \cdot A^{-1} = I$
(c) $A' \cdot A^{-1} = I$ (d) $A \cdot A' = I$

16. The rank of $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is equal to

(a) 4 (b) 3
(c) 5 (d) 1

17. $\begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$

(a) $x = 3, y = -1$ (b) $x = 2, y = 5$
(c) $x = 1, y = -1$ (d) $x = -1, y = 1$

18. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, which of the following result is true ?

(a) $A^2 = I$ (b) $A^2 = -I$
(c) $A^2 = 2I$ (d) None of these

19. With $1, \omega, \omega^2$ as cube roots of unity, inverse of which of the following matrices exists ?

(a) $\begin{bmatrix} 1 & \omega \\ \omega & \omega^2 \end{bmatrix}$ (b) $\begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix}$

(c) $\begin{bmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{bmatrix}$ (d) None of these

20. If A is an orthogonal matrix, then A^{-1} equals

(a) A (b) A'
(c) A^2 (d) none of these

21. If $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & -3 & 8 \\ 9 & 2 & 16 \end{bmatrix}$, then trace of A is

(a) 17 (b) 25
(c) 8 (d) 15

22. If A is a square matrix of order $n \times n$, then adj(adj A) is equal to

(a) $|A|^n A$ (b) $|A|^{n-1} A$
(c) $|A|^{n-2} A$ (d) $|A|^{n-3} A$

23. If A is a square matrix, then adj $A^T - (\text{adj } A)^T$ is equal to

(a) $2|A|$ (b) $2|A|I$
(c) null matrix (d) unit matrix

24. If $A = [a_{ij}]_{m \times n}$ is a matrix of rank r, then

(a) $r = \min(m, n)$ (b) $r > \min(m, n)$
(c) $r \leq \min(m, n)$ (d) none of these

25. If A is an orthogonal matrix, then

(a) $|A| = 0$ (b) $|A| = \pm 1$
(c) $|A| = \pm 2$ (d) none of these

26. The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{bmatrix}$ is

(a) idempotent (b) nilpotent
(c) involutory (d) orthogonal

27. Matrix theory was introduced by

(a) Cauchy-Riemann (b) Caley-Hamilton
(c) Newton (d) Cauchy-Schwar

28. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then A^{-1} is equal to

(a) $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ (b) $\begin{bmatrix} a^2 & 0 & 0 \\ 0 & ab & 0 \\ 0 & 0 & ac \end{bmatrix}$

(c) $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$ (d) $\begin{bmatrix} -a & 0 & 0 \\ 0 & -b & 0 \\ 0 & 0 & -c \end{bmatrix}$

29. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then adj A is equal to

(a) A (b) A^T
(c) $3A$ (d) $3A^T$

30. The matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is the matrix reflection in the line

(a) $x = 1$ (b) $x + y = 1$

(c) $y = 1$ (d) $x = y$

31. If I_n is the identity matrix of order n, then $(I_n)^{-1}$ is equal to

(a) does not exist (b) I_n

(c) 0 (d) nI_n

32. If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & x \end{bmatrix}$ is an idempotent matrix, then x is equal to

(a) −5 (b) −1

(c) −3 (d) −4

33. If A is non-singular matrix, then det (A^{-1}) is equal to

(a) $\det\left(\frac{1}{A^2}\right)$ (b) $\frac{1}{\det(A^2)}$

(c) $\det\left(\frac{1}{A}\right)$ (d) $\frac{1}{\det(A)}$

34. If A is a skew-symmetric matrix, then trace of A is

(a) −5 (b) 0

(c) 24 (d) 9

35. If the matrix $A = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i \\ -i & a \end{bmatrix}$, where $i = \sqrt{-1}$, is unitary, then a is equal to

(a) −2 (b) −1

(c) 0 (d) 1

36. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ and $AA' = I$, then $x + y$ is equal to

(a) −5 (b) −4

(c) −3 (d) −2

37. The sum of two idempotent matrices A and B is dempotent, if $AB = BA =$

(a) 4 (b) 3

(c) 2 (d) O

38. The rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is equal to

(a) 1 (b) 2

(c) 3 (d) none of these

39. If A, B, C are non-singular $n \times n$ matrices, then $(ABC)^{-1}$ is equal to

(a) $A^{-1}B^{-1}C^{-1}$ (b) $C^{-1}B^{-1}A^{-1}$

(c) $A^{-1}C^{-1}B^{-1}$ (d) $B^{-1}C^{-1}A^{-1}$

40. If $A^2 + A - I = O$, then A^{-1} is equal to

(a) $A - I$ (b) $I - A$

(c) $I + A$ (d) none of these

41. $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is symmetric, then x is equal to

(a) 3 (b) 5

(c) 2 (d) 4

42. Let a, b, c be positive real numbers. The following system of equations in x, y and z

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$, $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$,

$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ has

(a) no solution (b) unique solution

(c) infinitely many solutions (d) finitely many solutions

43. If the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{bmatrix}$ is singular, then λ is equal to

(a) 3 (b) 4

(c) 2 (d) 5

44. If A is a 3×3 matrix and $\det(3A) = k\{\det(A)\}$, then k is equal to

(a) 9 (b) 6

(c) 1 (d) 27

45. If $A = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(x)$, then A^{-1} is equal to

(a) $f(-x)$ (b) $f(x)$

(c) $-f(x)$ (d) $-f(-x)$

46. If A and B are square matrices of order 3 such that $|A| = -1, |B| = 3$, then $|3AB|$ equals

(a) −9 (b) −81

(c) −27 (d) 81

47. The equations $2x + y = 5$, $x + 3y = 5$, $x - 2y = 0$ have

(a) no solution (b) one solution

(c) two solutions (d) infinitely many solutions

48. If A is 3×4 matrix B is a matrix such $A'B$ and BA' are both defined, then B is of the type

(a) 3×4 (b) 3×3

(c) 4×4 (d) 4×3

49. If $A = [a \ \ b]$, $B = [-b \ \ -a]$ and $C = \begin{bmatrix} a \\ -a \end{bmatrix}$, then the correct statement is

(a) $A = -B$ (b) $A + B = A - B$

(c) $AC = BC$ (d) $CA = CB$

50. If $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$, then A^{-1} is equal to

(a) $\begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -2 \\ -3 & -5 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$

51. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to

(a) A (b) $-A$

(c) null matrix (d) I

52. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then the value of X^n is

(a) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ (b) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$

(c) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ (d) none of these

53. Matrix A such that $A^2 = 2A - I$, where I is the identity matrix. Then for $n \geq 2$, A^n is equal to

(a) $nA - (n-1)I$ (b) $nA - I$
(c) $2^{n-1}A - (n-1)I$ (d) $2^{n-1}A - I$

54. For the equations : $x + 2y + 3z = 1$, $2x + y + 3z = 2$, $5x + 5y + 9z = 4$

(a) there is only one solution
(b) there exists infinitely many solutions
(c) there is no solution
(d) none of the above

55. Consider the system of equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$, $a_3x + b_3y + c_3z = 0$. If

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0,$$

then the system has

(a) more than two solutions
(b) one trivial and one non-trivial solutions
(c) no solution
(d) only trivial solution (0, 0, 0)

Objective Questions Type II [One or more than one correct answer(s)]

In each of the questions below four choices of which one or more than one are correct. You have to select the correct answer(s) accordingly.

1. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then

(a) $A^3 = 9A$ (b) $A^3 = 27A$
(c) $A + A = A^2$ (d) A^{-1} does not exist

2. For all values of λ, the rank of the matrix

$$A = \begin{bmatrix} 1 & 4 & 5 \\ \lambda & 8 & 8\lambda - 6 \\ 1+\lambda^2 & 8\lambda + 4 & 2\lambda + 21 \end{bmatrix}$$

(a) for $\lambda = 2$, $\rho(A) = 1$ (b) for $\lambda = -1$, $\rho(A) = 2$
(c) for $\lambda \neq 2, -1$, $\rho(A) = 3$ (d) none of these

3. $\begin{bmatrix} 1 & -2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ is a

(a) rectangular matrix (b) singular matrix
(c) square matrix (d) non singular matrix

4. If A and B are square matrices of the same order such that $A^2 = A$, $B^2 = B$, $AB = BA = O$, then

(a) $(A + B)^2 = A + B$ (b) $AB^2 = O$
(c) $(A - B)^2 = A - B$ (d) none of these

5. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then

(a) $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \forall n \in N$ (b) $\lim\limits_{n \to \infty} \dfrac{A^{-n}}{n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$

(c) $\lim\limits_{n \to \infty} \dfrac{A^{-n}}{n^2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (d) none of these

6. The values of α for which the system of equations $x + y + z = 1$, $x + 2y + 4z = \alpha$, $x + 4y + 10z = \alpha^2$ is consistent, are given by

(a) 1, 2 (b) −1, 2
(c) 1, −2 (d) −1, −2

7. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where $bc \neq 0$) satisfies the equation $x^2 + k = O$, then

(a) $a + d = 0$ (b) $k = -|A|$
(c) $k = |A|$ (d) none of these

8. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then

(a) $A^2 - 4A - 5I_3 = O$ (b) $A^{-1} = \frac{1}{5}(A - 4I_3)$
(c) A^3 is not invertible (d) A^2 is invertible

9. Let $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then

(a) $A_{\alpha+\beta} = A_\alpha A_\beta$ (b) $A_\alpha^{-1} = A_{-\alpha}$
(c) $A_\alpha^{-1} = -A_\alpha$ (d) $A_\alpha^2 = -I$

10. If a square matrix $A = [a_{ij}]$, $a_{ij} = i^2 - j^2$ is of even order, then

(a) A is a skew matrix
(b) $|A|$ is a perfect square
(c) A is a symmetric and $|A| = 0$
(d) A is neither symmetric nor skew-symmetric

11. A matrix $A = [a_{ij}]_{m \times n}$ is

(a) a horizontal matrix if $m > n$
(b) a horizontal matrix if $m < n$
(c) a vertical matrix if $m > n$
(d) a vertical matrix if $m < n$

12. Which of the following is correct ?

(a) If A is a square matrix, $(A + A')$ is a symmetric matrix
(b) If A is a square matrix, $(A - A')$ is a skew symmetric matrix
(c) Every square matrix can be expressed as the sum of a symmetric and skew symmetric matrix
(d) Some elements of the skew symmetric matrix must be zero

13. If A and B are invertible square matrices of the same order, then which of the following is correct ?
(a) $\text{adj}(AB) = (\text{adj}B)(\text{adj}A)$
(b) $(\text{adj}A)' = (\text{adj}A')$
(c) $|\text{adj}A| = |A|^{n-1}$, where n is the order of matrix A
(d) $\text{adj}(\text{adj}B) = |B|^{n-2}B$, where n is the order of matrix B

14. If D_1 and D_2 are two 3×3 diagonal matrices, then
(a) D_1D_2 is a diagonal matrix
(b) $D_1^2 + D_2^2$ is a diagonal matrix
(c) $D_1D_2 = D_2D_1$
(d) D_1^n is a diagonal matrix $\forall\, n \in N$

15. Let A, B and C be 2×2 matrices with entries from the set of real numbers. Define o as follows
$AoB = \frac{1}{2}(AB + BA)$, then
(a) $AoB = BoA$
(b) $AoI = A$, I is an identity matrix of order 2
(c) $AoA = A^2$
(d) $Ao(B + C) = AoB + AoC$

16. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then
(a) $\text{adj}(\text{adj}A) = A$
(b) $|\text{adj}(\text{adj}A)| = 1$
(c) $|\text{adj}A| = 1$
(d) none of these

Linked-Comprehension Type

In these questions, a passage (paragraph) has been given followed by questions based on each of the passages. You have to answer the questions based on the passage given.

PASSAGE 1

Let A and B are two matrices of same order 3×3, where $A = \begin{pmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{pmatrix}$

On the basis of above information, answer the following questions :

1. If A is singular matrix , then $tr(A + B)$ is equal to
(a) 6 (b) 12 (c) 24 (d) 17

2. If matrix $2A + 3B$ is singular, then the value of 2λ is
(a) 11 (b) 13 (c) 15 (d) 17

3. If $\lambda = 3$, then $\frac{1}{7}(tr(AB) + tr(BA))$ is equal to
(a) 34 (b) 42 (c) 84 (d) 63

4. If $A = \begin{pmatrix} \lambda+2 & 3 & 1 \\ 8 & 4 & 2 \\ 10 & 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 2 & 3 \\ 5 & 2 & 3 \\ 4 & 5 & 2 \end{pmatrix}$, if $tr(A - 2B) = 0$, then the value of λ is
(a) 3 (b) 5
(c) 7 (d) 9

5. The correct statement is
(a) $(A + B)(A - B) = A^2 + B^2 - 2AB$
(b) $(A + B)^2 = A^2 + B^2 + AB + BA$
(c) $(A + B)^2 = A^2 + B^2 + 2AB$
(d) none of the above

PASSAGE 2

A and B are two matrices of same order 3×3, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{pmatrix}$$

On the basis of above information, answer the following questions :

1. The value of $\text{adj}(\text{adj}\,A)$ is equal to
(a) 2A (b) 4A (c) 8A (d) 16A

2. The value of $|\text{adj}(\text{adj}\,A)|$ is equal to
(a) 9 (b) 16 (c) 25 (d) 81

3. The value of $|\text{adj}\,B|$ is equal to
(a) 24 (b) 24^2 (c) 24^3 (d) 8^2

4. The value of $|\text{adj}\,(AB)|$ is equal to
(a) 24 (b) 24^2 (c) 24^3 (d) 65

5. The value of $|(\text{adj}(\text{adj}(\text{adj}(\text{adj}\,A))))|$ is equal to
(a) 2^4 (b) 2^9
(c) 2^{13} (d) 2^{19}

Matrix-Match Type

Given below are Matching Type Questions, with two columns (each having some items) each . Each item of Column I has to be matched with the items of Column II, by encircling the correct match(es).

NOTE An item of Column I can be matched with more than one items of Column II. All the items of Column II have to be matched.

1. Observe the following columns :

	Column I		Column II
(A)	If a, b and c are all different from zero such that $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$, then the matrix $A=\begin{pmatrix}1+a & 1 & 1\\ 1 & 1+b & 1\\ 1 & 1 & 1+c\end{pmatrix}$ is	(P)	symmetric
		(Q)	singular
(B)	If α, β, γ are three real numbers, then the matrix $A=\begin{pmatrix}1 & \cos(\alpha-\beta) & \cos(\alpha-\gamma)\\ \cos(\beta-\alpha) & 1 & \cos(\beta-\gamma)\\ \cos(\gamma-\alpha) & \cos(\gamma-\beta) & 1\end{pmatrix}$ is	(R)	non singular
		(S)	invertible
(C)	If A, B and C are the angles of a triangle, then the matrix $A=\begin{pmatrix}\sin 2A & \sin C & \sin B\\ \sin C & \sin 2B & \sin A\\ \sin B & \sin A & \sin 2C\end{pmatrix}$ is	(T)	non invertible

(A) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (B) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ (C) Ⓟ Ⓠ Ⓡ Ⓢ Ⓣ

2. Observe the following columns :

	Column I		Column II
(A)	If A, B and C be 2×2 matrices with entries from the set of real numbers. Define * as follows : $A * B=\frac{1}{2}(AB+BA)$, then	(P)	$A * B = B * A$
		(Q)	$A * (B + C) = A * B + A * C$
(B)	If A, B and C be 2×2 matrices with entries from the set of real numbers. Define * as follows : $A * B=\frac{1}{2}(AB'+A'B)$, then	(R)	$A * A = A^2$
		(S)	$A * I = A$
(C)	If A, B and C be 2×2 matrices with entries from the set of real numbers. Define * as follows : $A * B=\frac{1}{2}(AB-BA)$, then	(T)	$A * I = O$

Answers

Objective Questions Type I [Only one correct answer]

1. (a)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (c)
11. (b)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (b)
21. (d)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (b)	**28.** (c)	**29.** (d)	**30.** (d)
31. (b)	**32.** (c)	**33.** (d)	**34.** (b)	**35.** (b)	**36.** (c)	**37.** (d)	**38.** (a)	**39.** (b)	**40.** (c)
41. (b)	**42.** (b)	**43.** (a)	**44.** (d)	**45.** (a)	**46.** (b)	**47.** (b)	**48.** (a)	**49.** (c)	**50.** (b)
51. (d)	**52.** (d)	**53.** (a)	**54.** (a)	**55.** (a)					

Objective Questions Type II [One or more than one correct answer(s)]

1. (a, d)	**2.** (a, b, c)	**3.** (c, d)	**4.** (a, b)	**5.** (a, b, c)
6. (a, c)	**7.** (a, c)	**8.** (a, b, d)	**9.** (a, b)	**10.** (a, b)
11. (b, c)	**12.** (a, b, c, d)	**13.** (a, b, c, d)	**14.** (a, b, c, d)	**15.** (a, b, c, d)
16. (a, b, c)				

Linked-Comprehension Type

Passage 1 **1.** (c) **2.** (d) **3.** (a) **4.** (c) **5.** (b)   **Passage 2** **1.** (a) **2.** (b) **3.** (b) **4.** (b) **5.** (c)

Numerical Grid-Based Problems

1. 1 2 9 6 **2.** 0 0 0 1 **3.** 0 8 9 6 **4.** 1 0 0 0

5. 0 1 4 4 **6.** 0 2 5 6 **7.** 1 4 5 3 **8.** 2 0 0 8

Matrix-Match Type

1. A→ (P, R, S); B→ (P, Q, T); C→ (P, Q, T)   **2.** A→ (P, Q, R, S); B→ (P, Q); C→ (Q, T)